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SYSTEMATIC ERROR ON WEAK PHASE γ FROM $B \rightarrow \pi^+\pi^-$ AND $B \rightarrow K\pi$.¹

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When CP asymmetries in $B^0(t) \rightarrow \pi^+\pi^-$ are combined using broken flavor SU(3) with decay rates for $B^+ \rightarrow K^0\pi^+$ and/or $B^0 \rightarrow K^+\pi^-$, one can obtain stringent limits on the weak phase γ which are consistent with those obtained from other CKM constraints. Experimental data in the past few years have improved to the extent that systematic errors associated with uncertainty in SU(3) symmetry breaking dominate the determination of γ . We obtain a value $\gamma = (73 \pm 4_{-8}^{+10})^\circ$, where the first error is statistical while the second one is systematic.

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Time-dependent CP-violating asymmetries in the decays $B^0(t) \rightarrow \pi^+\pi^-$ and their charge conjugates can provide useful information on the weak phase α or γ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. These quantities $C_{\pi\pi} \equiv -A_{CP}(B^0 \rightarrow \pi^+\pi^-)$ and $S_{\pi\pi}$, defined by [1]

$$\frac{\Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) - \Gamma(B^0(t) \rightarrow \pi^+\pi^-)}{\Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) + \Gamma(B^0(t) \rightarrow \pi^+\pi^-)} = -C_{\pi\pi} \cos(\Delta mt) + S_{\pi\pi} \sin(\Delta mt), \quad (1)$$

are respectively $C_{\pi\pi} = 0$ and $S_{\pi\pi} = -\sin(2\alpha)$ in the limit in which a single “tree” amplitude, as shown in Fig. 1 (a), dominates the $B^0 \rightarrow \pi^+\pi^-$ decay. The two asymmetries are modified by a contribution from the “penguin” amplitude [Fig. 1 (b)] [1, 2]. The theoretically most precise method for determining γ in the presence of a penguin amplitude is based on applying an isospin analysis to all three $B \rightarrow \pi\pi$ decay modes and their charge-conjugates [3]. The current precision of this method, limited by a sizable decay rate for $B^0 \rightarrow \pi^0\pi^0$ and by a large experimental error in $A_{CP}(\pi^0\pi^0)$ [4], does not permit a complete construction of the two isospin triangles describing $B \rightarrow \pi\pi$ and $\bar{B} \rightarrow \pi\pi$ amplitudes. Thus, the three $B \rightarrow \pi\pi$ decay rates and corresponding direct CP asymmetries lead to a rather large systematic error of $\pm 16^\circ$ in γ [5].

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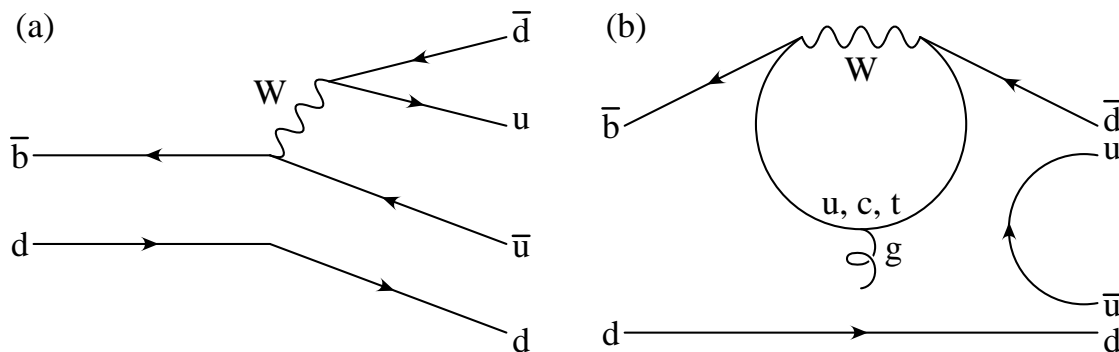


Figure 1: Examples of decay topologies for $B^0 \rightarrow \pi^+ \pi^-$. (a) Tree; (b) penguin.

An alternative way of treating a subsidiary penguin amplitude in $B^0 \rightarrow \pi^+ \pi^-$ is by estimating its contribution with the help of flavor SU(3) and the decays $B \rightarrow K\pi$, which are dominated by a $b \rightarrow s$ penguin contribution [6]. In this approach, a theoretical uncertainty in estimating the penguin amplitude is translated into a smaller relative error in γ because of the subdominant nature of a penguin amplitude in $B^0 \rightarrow \pi^+ \pi^-$. This suppression of error is expected to be somewhat less effective here than in $B^0 \rightarrow \rho^+ \rho^-$, where a smaller penguin contribution estimated using $B^+ \rightarrow K^{*0} \rho^+$ has been shown to lead to a systematic error of only several degrees in γ [7], smaller than the error associated with an isospin analysis in $B \rightarrow \rho\rho$ [5].

In Ref. [8] we demonstrated the precision on α obtained when deriving the penguin amplitude in $B^0 \rightarrow \pi^+ \pi^-$ either from $B^+ \rightarrow K^0 \pi^+$ (a pure-penguin process) or $B^0 \rightarrow K^+ \pi^-$ (a process with a small tree contribution). Data available in May 2004 from BaBar [9] and Belle [10],

$$C_{\pi\pi} = \begin{cases} -0.19 \pm 0.19 \pm 0.05, \\ -0.58 \pm 0.15 \pm 0.07, \end{cases} \quad S_{\pi\pi} = \begin{cases} -0.40 \pm 0.22 \pm 0.03, \\ -1.00 \pm 0.21 \pm 0.07, \end{cases} \quad \begin{matrix} \text{BaBar,} \\ \text{Belle,} \end{matrix} \quad (2)$$

implied averages

$$C_{\pi\pi} = -0.46 \pm 0.13, \quad S_{\pi\pi} = -0.74 \pm 0.16. \quad (3)$$

Using these data and extracting the penguin amplitude from $B^0 \rightarrow K^+ \pi^-$ (just slightly more restrictive than using $B^+ \rightarrow K^0 \pi^+$), we found that $\alpha = (103 \pm 17)^\circ$ or $\alpha = (107 \pm 13)^\circ$, depending on SU(3)-breaking factors. With the current value of $\beta = (21.3 \pm 1.0)^\circ$ obtained from CP asymmetries dominated by the subprocess $b \rightarrow c\bar{c}s$ [4], this would entail $\gamma = (56 \pm 17)^\circ$ or $\gamma = (52 \pm 13)^\circ$. We anticipated that reduction of the errors by a factor of two would not present difficulties.

The experimental data have improved significantly in the past few years. Asymmetries reported recently by BaBar [11] and Belle [12],

$$C_{\pi\pi} = \begin{cases} -0.21 \pm 0.09 \pm 0.02, \\ -0.55 \pm 0.08 \pm 0.05, \end{cases} \quad S_{\pi\pi} = \begin{cases} -0.60 \pm 0.11 \pm 0.03, \\ -0.61 \pm 0.10 \pm 0.04, \end{cases} \quad \begin{matrix} \text{BaBar,} \\ \text{Belle,} \end{matrix} \quad (4)$$

imply averages [4]

$$C_{\pi\pi} = -0.38 \pm 0.07, \quad S_{\pi\pi} = -0.61 \pm 0.08, \quad (5)$$

Table I: Old and new branching ratios for $B \rightarrow \pi^+\pi^-$ and $B \rightarrow K\pi$ (in units of 10^{-6}).

Year	$B^0 \rightarrow \pi^+\pi^-$	$B^+ \rightarrow K^0\pi^+$	$B^0 \rightarrow K^+\pi^-$
2004	4.6 ± 0.4	21.8 ± 1.4	18.2 ± 0.8
2007	5.16 ± 0.22	23.1 ± 1.0	19.4 ± 0.6

with errors only about half the size of errors in (3). A similar reduction of errors by a factor two occurred in ratios of $B \rightarrow K\pi$ to $B \rightarrow \pi^+\pi^-$ branching ratios affecting the extraction of γ [see Eq. (22) below.] Old and new charge-averaged branching ratios for these processes, in units of 10^{-6} , are tabulated in Table I.

The purpose of the present note is to use the improved data for obtaining γ with an experimental error, and to confront a systematic theoretical error in γ related to patterns of flavor SU(3) symmetry breaking. We shall update our analysis of Ref. [8], using patterns for SU(3) breaking which differ by $\pm\mathcal{O}(20\%)$, quote the associated systematic uncertainty in γ , and compare with a contemporary analysis [13].

The reader may consult Refs. [6, 8] for earlier references and notation. We recapitulate the main formulae for $C_{\pi\pi}$ and $S_{\pi\pi}$. We integrate out the top-quark contribution in the $b \rightarrow d$ penguin transition and use unitarity of the CKM matrix. Absorbing a P_{tu} term in the tree amplitude T , one writes

$$A(B^0 \rightarrow \pi^+\pi^-) = T e^{i\gamma} + P e^{i\delta} . \quad (6)$$

The tree T and penguin P amplitudes, which involve magnitudes of CKM factors, $|V_{ub}^*V_{ud}|$ and $|V_{cb}^*V_{cd}|$, are taken to be real and positive and the strong phase δ is taken to lie in the range $-\pi \leq \delta \leq \pi$. For $\bar{B}^0 \rightarrow \pi^+\pi^-$, $\gamma \rightarrow -\gamma$. The asymmetries $C_{\pi\pi}$ and $S_{\pi\pi}$ are given by [1]

$$C_{\pi\pi} \equiv \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} , \quad S_{\pi\pi} \equiv \frac{2\text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} , \quad (7)$$

where

$$\lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{A(\bar{B}^0 \rightarrow \pi^+\pi^-)}{A(B^0 \rightarrow \pi^+\pi^-)} . \quad (8)$$

Substituting (6), one obtains [6]

$$C_{\pi\pi} = \frac{2r \sin \delta \sin(\beta + \alpha)}{R_{\pi\pi}} , \quad (9)$$

$$S_{\pi\pi} = \frac{\sin 2\alpha + 2r \cos \delta \sin(\beta - \alpha) - r^2 \sin 2\beta}{R_{\pi\pi}} , \quad (10)$$

$$R_{\pi\pi} \equiv 1 - 2r \cos \delta \cos(\beta + \alpha) + r^2 , \quad (11)$$

where $r \equiv P/T$ is a ratio of penguin to tree amplitudes.

In the absence of a penguin amplitude ($r = 0$) one has $C_{\pi\pi} = 0$, $S_{\pi\pi} = \sin 2\alpha$. To first order in r , one finds

$$C_{\pi\pi} = 2r \sin \delta \sin(\beta + \alpha) + \mathcal{O}(r^2) , \quad (12)$$

$$S_{\pi\pi} = \sin 2\alpha + 2r \cos \delta \sin(\beta + \alpha) \cos 2\alpha + \mathcal{O}(r^2) , \quad (13)$$

so that in the linear approximation the allowed region in the $(S_{\pi\pi}, C_{\pi\pi})$ plane is confined to an ellipse centered at $(\sin 2\alpha, 0)$, with semi-principal axes $2[r \sin(\beta + \alpha) \cos 2\alpha]_{\max}$ and $2[r \sin(\beta + \alpha)]_{\max}$. We will use the exact expressions (9)–(11).

Given a value of β , as already measured in $B^0(t) \rightarrow J/\psi K_S$ [4], the observables $C_{\pi\pi}$ and $S_{\pi\pi}$ provide two equations for α or γ , r , and δ . At least one additional constraint is needed to determine α or γ .

The $B \rightarrow K\pi$ decay amplitudes are described in terms of primed quantities, T' and P' [14]. We introduce an SU(3) breaking factor f_K/f_π in tree amplitudes assuming that these amplitudes factorize [15] [see discussion two paragraphs below Eq. (23)], but begin by assuming an arbitrary SU(3)-breaking factor ξ_P in determining P' from P , as factorization is not expected to hold for penguin amplitudes [16, 17]:

$$T' = \frac{f_K}{f_\pi} \frac{V_{ub}^* V_{us}}{V_{ub}^* V_{ud}} T = \frac{f_K}{f_\pi} \bar{\lambda} T , \quad P' = \xi_P \frac{V_{cb}^* V_{cs}}{V_{cb}^* V_{cd}} P = -\xi_P \bar{\lambda}^{-1} P . \quad (14)$$

Here

$$\bar{\lambda} \equiv \frac{\lambda}{1 - \lambda^2/2} = 0.230 . \quad (15)$$

Contributions of amplitudes involving the spectator quark are expected to be suppressed by Λ_{QCD}/m_b relative to those considered [14, 17]. This includes exchange and penguin annihilation amplitudes ($E + PA$) in $B^0 \rightarrow \pi^+\pi^-$ and an annihilation amplitude (A) in $B^+ \rightarrow K^0\pi^+$. Evidence for small $E + PA$ is provided by an upper limit on $\mathcal{B}(B^0 \rightarrow K^+K^-)$ [4, 18]. We will neglect these contributions, but will include the effect of A in the systematic error. In this approximation one may express $B \rightarrow K\pi$ amplitudes in terms of those contributing to $B^0 \rightarrow \pi^+\pi^-$:

$$A(B^+ \rightarrow K^0\pi^+) = -\xi_P \bar{\lambda}^{-1} P e^{i\delta} , \quad (16)$$

$$A(B^0 \rightarrow K^+\pi^-) = -\frac{f_K}{f_\pi} \bar{\lambda} T e^{i\gamma} + \xi_P \bar{\lambda}^{-1} P e^{i\delta} . \quad (17)$$

The CP asymmetry in the first process vanishes, while that of $B^0 \rightarrow K^+\pi^-$

$$\Gamma(\bar{B}^0 \rightarrow K^-\pi^+) - \Gamma(B^0 \rightarrow K^+\pi^-) = -\xi_P \frac{f_K}{f_\pi} [\Gamma(\bar{B}^0 \rightarrow \pi^+\pi^-) - \Gamma(B^0 \rightarrow \pi^+\pi^-)] . \quad (18)$$

is related to the asymmetry in $B^0 \rightarrow \pi^+\pi^-$ [19, 20],

Each of the two charge averaged rates $\bar{\Gamma}(B^+ \rightarrow K^0\pi^+) \equiv [\Gamma(B^+ \rightarrow K^0\pi^+) + \Gamma(B^- \rightarrow \bar{K}^0\pi^-)]/2$ and $\bar{\Gamma}(B^0 \rightarrow K^+\pi^-) \equiv [\Gamma(B^0 \rightarrow K^+\pi^-) + \Gamma(\bar{B}^0 \rightarrow K^-\pi^+)]/2$ provides an additional constraint on the three parameters r , δ and α . Normalizing these rates by $\bar{\Gamma}(B^0 \rightarrow \pi^+\pi^-) \equiv [\Gamma(B^0 \rightarrow \pi^+\pi^-) + \Gamma(\bar{B}^0 \rightarrow \pi^+\pi^-)]/2$, we define two ratios

$$\mathcal{R}_+ \equiv \frac{\bar{\lambda}^2 \bar{\Gamma}(B^+ \rightarrow K^0\pi^+)}{\bar{\Gamma}(B^0 \rightarrow \pi^+\pi^-)} , \quad \mathcal{R}_0 \equiv \frac{\bar{\lambda}^2 \bar{\Gamma}(B^0 \rightarrow K^+\pi^-)}{\bar{\Gamma}(B^0 \rightarrow \pi^+\pi^-)} , \quad (19)$$

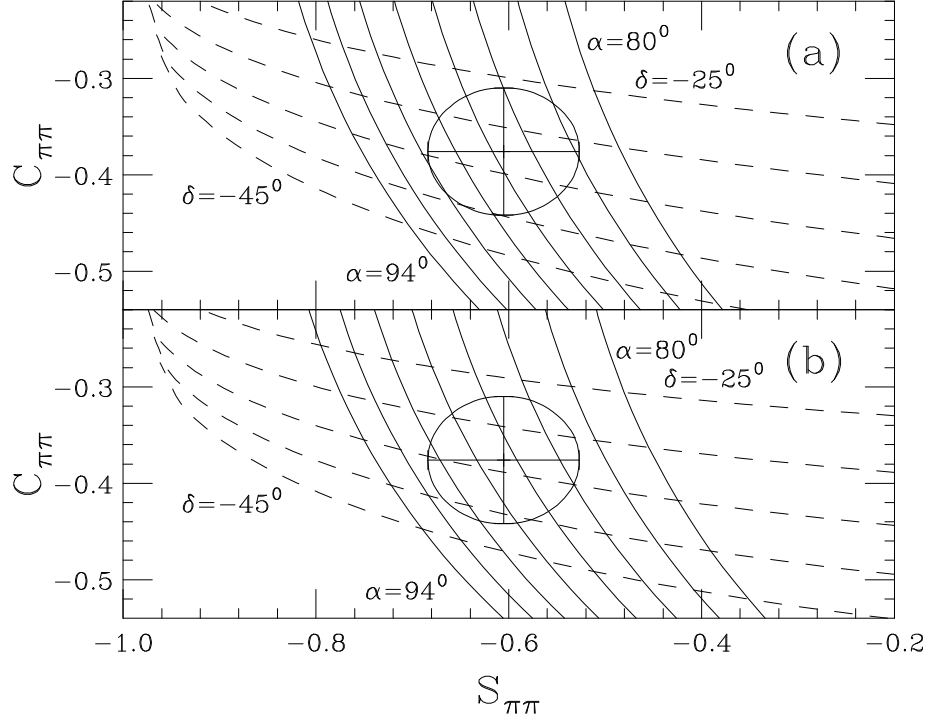


Figure 2: Values of $C_{\pi\pi}$ plotted against $S_{\pi\pi}$ for values of α spaced by 2 degrees (solid curves) and δ spaced by 5 degrees (dashed contours), with a parameter $\xi_P = 1$ describing the degree of SU(3) violation in the ratio P'/P . The degree of penguin “pollution” is estimated in (a) from $B^+ \rightarrow K^0 \pi^+$ and in (b) from $B^0 \rightarrow K^+ \pi^-$.

given by

$$\mathcal{R}_+ = \frac{\xi_P^2 r^2}{R_{\pi\pi}}, \quad (20)$$

$$\mathcal{R}_0 = \frac{\xi_P^2 r^2 + 2\xi_P \bar{\lambda}'^2 \cos \delta \cos(\beta + \alpha) + \bar{\lambda}'^4}{R_{\pi\pi}}, \quad \bar{\lambda}' \equiv \sqrt{\frac{f_K}{f_\pi}} \bar{\lambda}. \quad (21)$$

Using branching ratios in Table I and the lifetime ratio [4] $\tau(B^+)/\tau(B^0) = 1.076 \pm 0.008$, one finds the following values for \mathcal{R}_+ and \mathcal{R}_0 ,

$$\mathcal{R}_+ = 0.220 \pm 0.013, \quad \mathcal{R}_0 = 0.199 \pm 0.010, \quad (22)$$

As mentioned, the 5% errors here are half those quoted in Ref. [8]. Here as in Ref. [6] we have applied small corrections for phase space factors.

Eqs. (9)-(11) and either (20) or (21) provide three equations for r, α and δ , for given β and for a given SU(3)-breaking parameter ξ_P describing the ratio of $\Delta S = 1$ and $\Delta S = 0$ penguin amplitudes. Eq. (20) or (21) may be used to eliminate r . Thus, in Figs. 2, 3 and 4 we plot $C_{\pi\pi}$ and $S_{\pi\pi}$ as functions of α and δ for three values of the SU(3) breaking parameter, $\xi_P = 1$, $\xi_P = f_K/f_\pi = 1.22$, and $\xi_P = 0.79$. The latter is the central value of $\xi_P = 0.79 \pm 0.18$, obtained by solving Eq. (18) using $B^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ \pi^-$ branching ratios from Table I, the value of $C_{\pi\pi}$ in (5), and $A_{CP}(B^0 \rightarrow K^+ \pi^-) = -0.097 \pm 0.012$ [4].

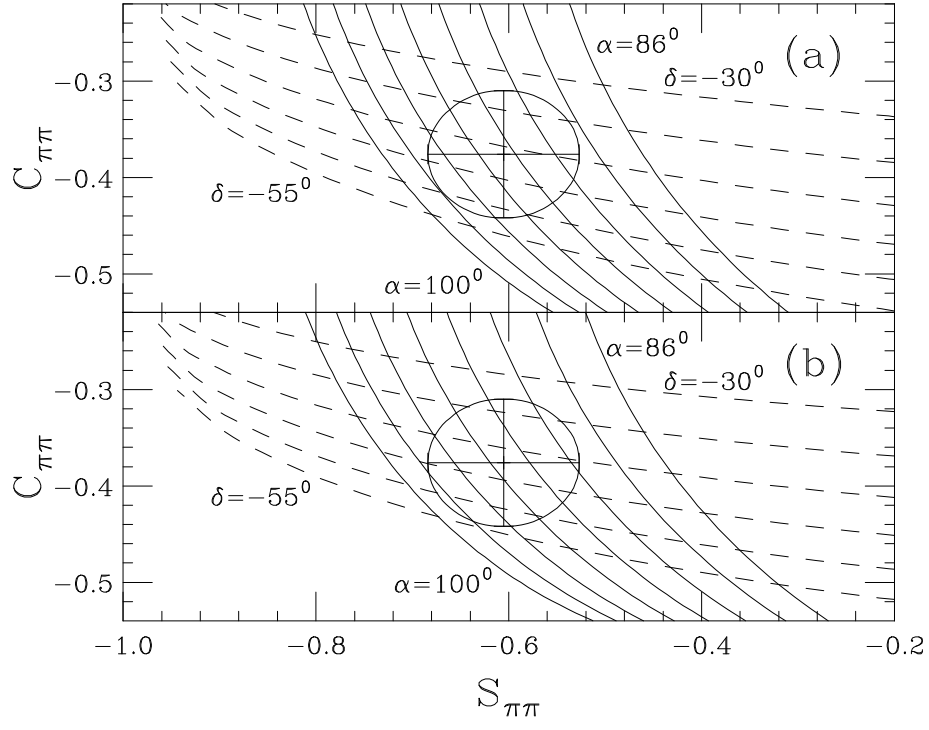


Figure 3: Same as Fig. 2 but with $\xi_P = f_K/f_\pi = 1.22$.

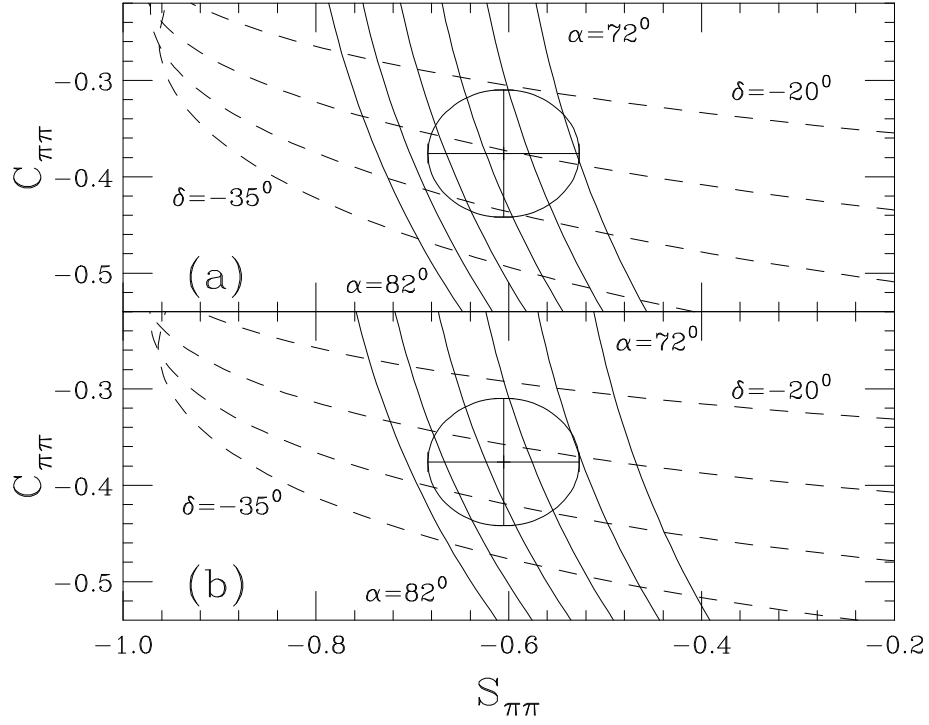


Figure 4: Same as Fig. 2 but with $\xi_P = 0.79$.

The error ellipses in Figs. 2, 3 and 4 describing the measurements (5) encompass somewhat different ranges for α (or γ) and δ . The three corresponding pairs of ranges are

$$\begin{array}{lll}
\text{Fig. 2 } (\xi_P = 1) & \text{Fig. 3 } (\xi_P = 1.22) & \text{Fig. 4 } (\xi_P = 0.79) \\
81^\circ \leq \alpha \leq 91^\circ & 88^\circ \leq \alpha \leq 99^\circ & 72^\circ \leq \alpha \leq 81^\circ \\
(68^\circ \leq \gamma \leq 78^\circ) & (60^\circ \leq \gamma \leq 71^\circ) & (78^\circ \leq \gamma \leq 87^\circ) \\
-42^\circ \leq \delta \leq -26^\circ & -54^\circ \leq \delta \leq -32^\circ & -32^\circ \leq \delta \leq -20^\circ .
\end{array} \tag{23}$$

Here we have taken in each figure the union of the regions allowed by constraints (a) and (b) from \mathcal{R}_+ and \mathcal{R}_0 rather than their intersection. The small differences between the values following from these two constraints, at the level of a degree or two, should be included in the systematic rather than statistical errors. These differences may be associated with neglecting an annihilation amplitude in the ratio \mathcal{R}_+ .

As in Ref. [13], we find a very small statistical error in γ of only 4 degrees. The systematic error in γ associated with uncertainty in SU(3) breaking is larger. The change from $\xi_P = 1$ to $\xi_P = 1.22$ and $\xi_P = 0.79$ shifts γ down by 8° and up by 10° , respectively. Similarly, under these changes δ becomes more negative by about 10° and less negative by about 8° , respectively.

We now discuss some additional possible sources of systematic error. They all indicate that the range we quote for systematic errors is probably conservative.

(1) Because we have absorbed a P_{tu} term in the tree amplitude T , as noted above Eq. (6), one might question the applicability of factorization to the estimate (14) of $T'/T = \bar{\lambda}f_K/f_\pi$. We have investigated the effect of replacing f_K/f_π in this expression by a parameter ξ_T with range similar to that allowed for ξ_P . We find very little dependence on ξ_T , with variations between 0.79 and 1.22 leading to variations of α and δ of at most a degree or two. This may be seen from Eq. (21) with $\bar{\lambda}^2$ replaced by $\xi_T \bar{\lambda}^2$. The second term in the numerator, proportional to r and ξ_T , is much smaller than the first, proportional to r^2 and independent of ξ_T . For a reasonable value of $r \sim 0.4$ – 0.5 and for $90^\circ < \beta + \alpha < 120^\circ$, one has $r^2 \sim 0.2$ while $2r\xi_T \bar{\lambda}^2 |\cos(\beta + \alpha)| < 0.03$. The third term in the numerator, $\xi_T^2 \bar{\lambda}^4$, is negligible.

(2) The determinations of α and δ in which the penguin pollution in $B^0 \rightarrow \pi^+\pi^-$ is obtained from the decay $B^+ \rightarrow K^0\pi^+$ via Eq. (20) are trivially independent of ξ_T , as they do not require the estimate of T' at all. It is then reassuring that they are consistent within a degree or two with those obtained from Eq. (21).

(3) The relation (18) between partial width differences now becomes

$$\Gamma(\bar{B}^0 \rightarrow K^-\pi^+) - \Gamma(B^0 \rightarrow K^+\pi^-) = -\xi_P \xi_T [\Gamma(\bar{B}^0 \rightarrow \pi^+\pi^-) - \Gamma(B^0 \rightarrow \pi^+\pi^-)] \tag{24}$$

and with the observed values of branching ratios and CP asymmetries may be used to constrain the product

$$\xi_P \xi_T = 0.96 \pm 0.18 \tag{25}$$

Indeed, the case illustrated in Fig. 3, discussed in Ref. [13], violates these bounds, and is only viable if, as in that work, one favors the BaBar result [11] implying a smaller direct asymmetry in $B^0 \rightarrow \pi^+\pi^-$.

(4) One can extrapolate beyond the values of ξ_P shown in Figs. 2–4 if desired. The upper and lower limits on γ are shown for a range of ξ_P and the nominal value $\xi_T = 1.22$ in Fig. 5. The lower limit $\xi_P \geq 0.64$ is based on the 1σ lower limit of the

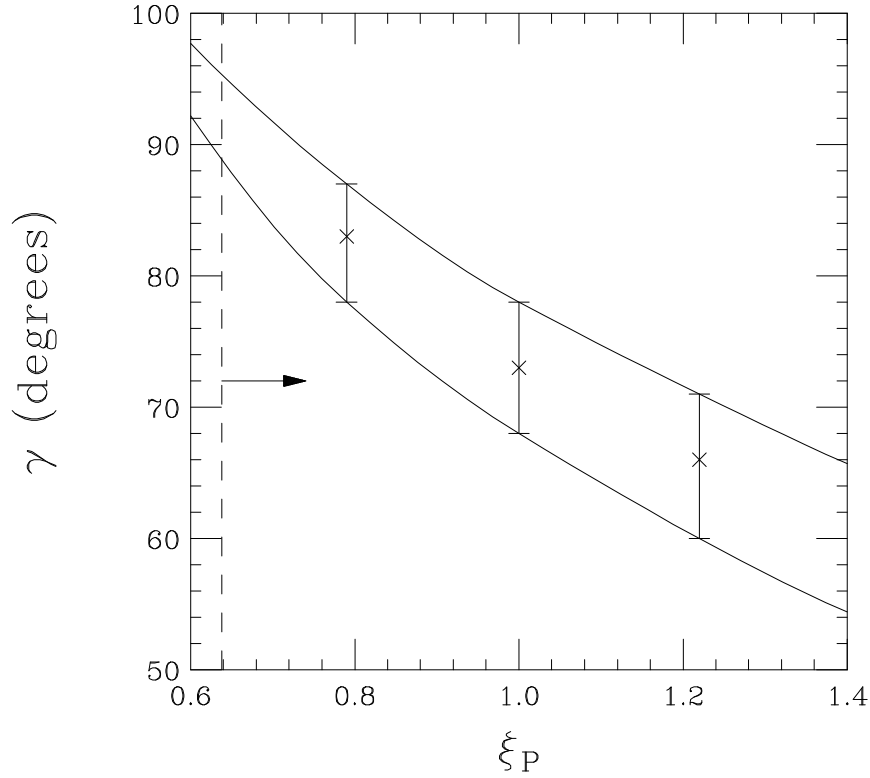


Figure 5: Dependence on ξ_P of upper and lower limits on γ . The three cases discussed in Figs. 2–4 are shown as plotted points. The dashed vertical line corresponds to the lower limit $\xi_P \geq 0.64$ discussed in the text.

constraint $\xi_P \xi_T = 0.96 \pm 0.18$ for $\xi_T = 1.22$. It implies only the rather weak bound $\gamma \leq 95^\circ$. However, even the choice $\xi_P = 0.79$ would suggest that SU(3) breaking acts in opposite ways for the tree and penguin, an unlikely circumstance given the tendencies of form factors involving final-state strange quarks to be enhanced relative to those involving nonstrange quarks.

(5) The neglect of possible E and PA contributions is an approximation based on theoretical estimates which can only be fully justified once the branching ratio for $B^0 \rightarrow K^+ K^-$ has been shown to lie definitively below 10^{-7} , as we have emphasized in several previous references (see, e.g., [18]). The present upper limit is about three times this value [4]. One should not take the unexpectedly large branching ratio for $B^0 \rightarrow \pi^0 \pi^0$ as evidence for large $E + PA$, as it can be explained by a larger-than-expected contribution from the color-suppressed tree amplitude C [21].

To summarize, the time-dependent asymmetries in $B^0 \rightarrow \pi^+ \pi^-$ have realized their statistical potential in pinning down weak phases, implying $\alpha = (86 \pm 4_{-10}^{+8})^\circ$, $\gamma = (73 \pm 4_{-8}^{+10})^\circ$. The relative strong phase δ between penguin and tree amplitudes is found to be $\delta = (-33 \pm 7_{-10}^{+8})^\circ$. The systematic errors quoted here are those associated with likely uncertainties in flavor-SU(3) breaking. Under exceptional circumstances (such as an anomalously small non-strange penguin amplitude) the systematic errors could even exceed those quoted. In order to add useful information to this

largely model-independent discussion, explicit theoretical calculations such as QCD-factorization [15], Soft Collinear Effective Theory (SCET) [17] or Perturbative QCD (pQCD) [22] need to predict δ with an accuracy better than the systematic error of approximately $\pm 10^\circ$ found above.

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